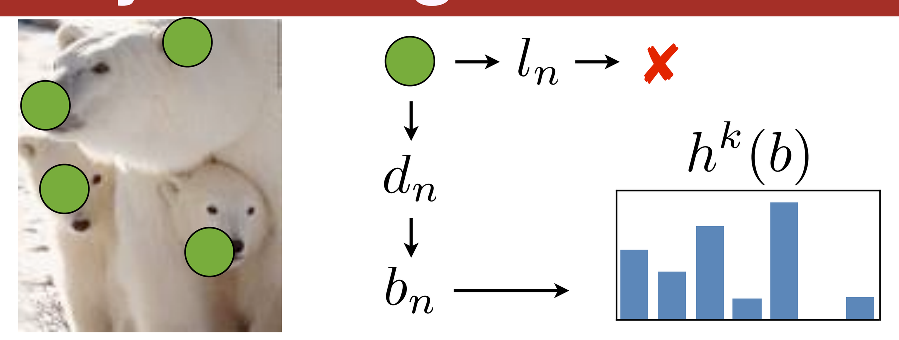
Relaxed Matching Kernels

Andrea Vedaldi and Stefano Soatto - UCLA Vision Lab

Kernels for Recognition

Goal: capture systematically recent **kernels for bag-of-features** representations which exploit spatial information. Introduce **new kernels.**

Beyond Bag of Features



- **Visual features**
  - Locations $l_1, \ldots, l_N \in \mathbb{R}^2$
  - Descriptors $d_1, \ldots, d_N \in \mathcal{F}$
  - Quantization (visual words) $b_1, \ldots, b_N \in B$
- **Bag-of-features**
  histograms of visual words: $h^k(b), k = 1, 2$
- **Comparison**
  $$K(h^1, h^2) = \sum_{b \in B} \min\{h^1(b), h^2(b)\}$$

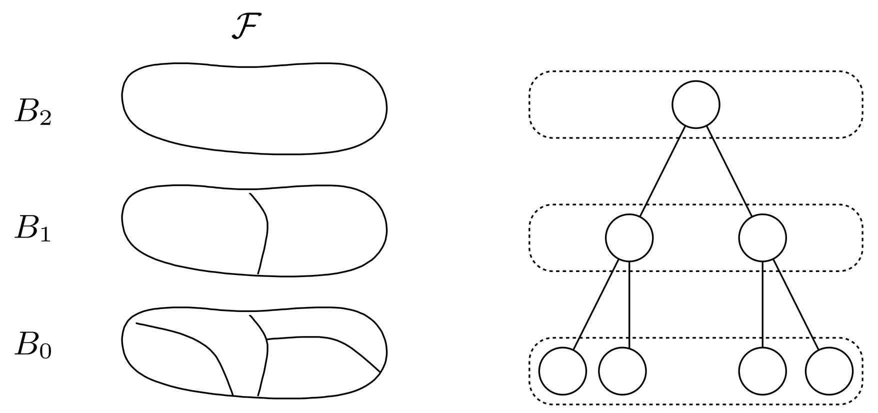
Beyond bag-of-features:

- Pyramid Matching Kernels (PMK). Hierarchical visual words
- Spatial Pyramid Matching Kernels (SPMK). Locations + descriptors → visual words
- Proximity Distribution Kernels (PDK). Pairs of nearby descriptors → visual words

Relaxed Matching Kernels

Relaxed Matching Kernels (RMK) generalize PMK, SPMK and PDK. They also include many other kernels. Basic idea:

- No optimal quantization
- Consider multiple, hierarchical quantizations



- **Multiple quantizations**
  - **Tree:** hierarchy of visual words
  - **Relaxation:** tree cut
- **Comparison**
  - similarity score at level *r*: $F_r = K(h_r^1, h_r^2)$
  - overall similarity $$K(I^1, I^2) = \sum_{r=0}^{R-1} w_r F_r$$

On the Base Kernel

How do we choose the base kernel $K(h^1, h^2)$?

[19] introduced a large family of kernels for probability distributions that can readily be used in the RMK framework.

$$K(h_r^1, h_r^2) = \sum_{b \in B_r} k(h_r^1(b), h_r^2(b))$$

- L1 kernel $k(a, b) = \min\{a, b\}$
- Chi2 kernel $k(a, b) = 2(ab)/(a + b)$
- Hellinger's kernel $k(a, b) = \sqrt{ab}$

Radial Basis Function versions of all RMKs are defined up to a scaling parameter

$$K_{\text{RBF}}(I^1, I^2) = \exp\{-\lambda K(I^1, I^2)\}$$

Lemma: All such base kernels yield **positive definite** (PD) RMKs. The RBF versions are PD as well.

On the Weights

- Are relaxations redundant?
- Are we double-counting features?
- What is the meaning of the weights?

Theorem. F_r is a non decreasing function of the relaxation order *r* for all choices of the base kernel.

F_r can be thought as a distribution over relaxations and an RMK as the expected values of the weights w_r. RMKs can also be rewritten as:

$$K(I^1, I^2) = \sum_{r=0}^{R-1} w_r F_r = \sum_{r=0}^{R-1} (W_{R-1} - W_{r-1}) f_r$$

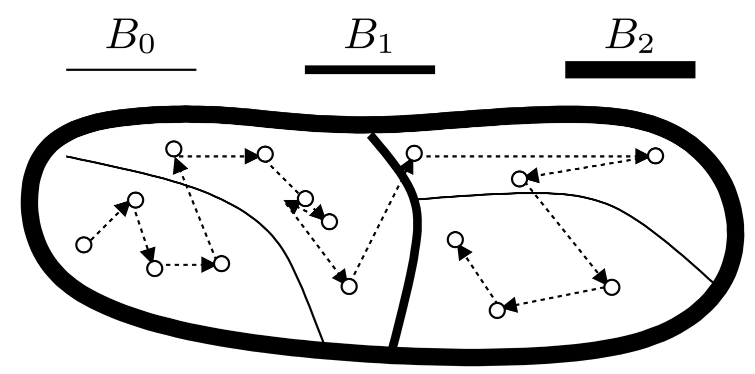
- $f_r = F_r - F_{r-1}$ is the variation of the similarity score at level r
- $W_r = \sum_{q=0}^{r} w_q$ are the integral weights
- $W_{R-1} - W_{r-1}$ decreases monotonically to zero

Interpretation: An RMK searches for the smaller relaxation order for which the data match well.

Efficient Calculation

All RMKs can be efficiently computed by a single pass on through finest quantization level.

Key idea: Visit bins by traversing all visual words once. This is possible because visual words are organized hierarchically.



New RMKs

Graph Matching Kernels (GMK). Features are often arranged in graphical configurations. GMKs compare graphs of visual words which match coarsely.

- Features: pairs of visual words at graph distance less than *r.*
- Matching: count how many similar pairs there are.

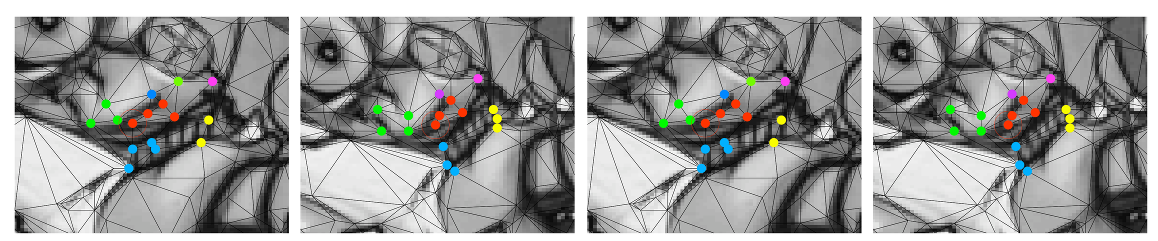
$$F_r = \sum_{(d_i, d_j, \rho) \in B_r} k(h_r^1(d_i, d_j, \rho), h_r^2(d_i, d_j, \rho))$$

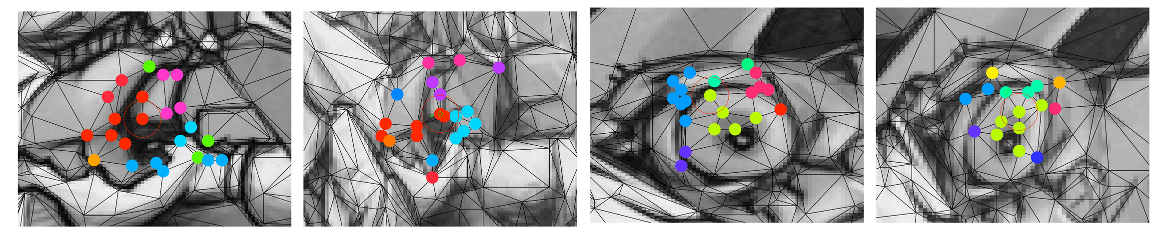
Observation: If the nodes have unique names (visual words), then a GMK is zero if, and only if, the graphs are identical.

Agglomerative Information Bottleneck Kernels (AIBMK). Similar to PMK, but it creates hierarchy based on AIB.

Experiments

GMK for matching graphlets of features.

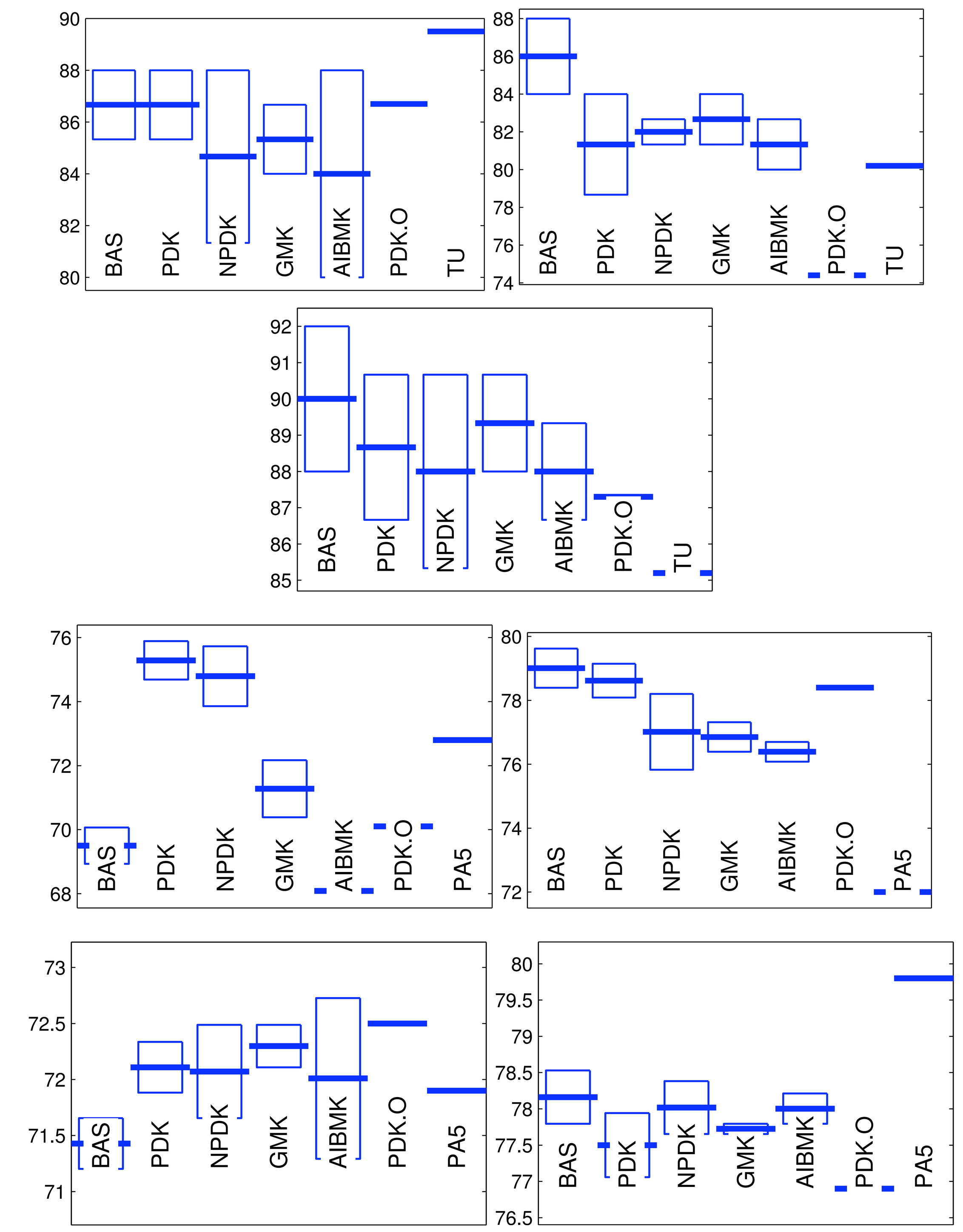




- Test robustness of in graph matching. Graphs: Delaunay Triangulation, SIFT features at vertices coarsely quantized
- Robust matching up to 40-50 degrees.

RMKs for object categorization.

- BAS: Baseline Bag-of-features method
- PDK: Proximity Distribution Kernel
- NPDK: Normalized PDK
- GMK: Graph Matching Kernel
- AIBMK: Agglo. Info. Kernel
- PDK.O: PDK (our implementation)
- PA5: Pascal05 Winner
- TU: Tuytelaars 07



In order: Graz-02 Bikes, Cars, People, Pascal-05 Bikes, Cars, People, Motorbikes

Conclusions

- RMKs generalize previous matching kernels for image comparison.
- RMKs highlight common properties and provide an universal algorithm.
- Careful experimentation reveals that current formulations may be insufficient to exploit spatial information.